Applications of Mathematical Ecology to Rotational Grazing

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Outline

- Rotational grazing is known to be more productive than continuous grazing as described in agricultural publications
- Provides grasses with more sunlight, water, and nutrients, thus allowing the roots to deepen
- Increases quality and quantity of forage for a higher number of cattle sustainable and less supplemental feed (University of Kentucky, 2011)

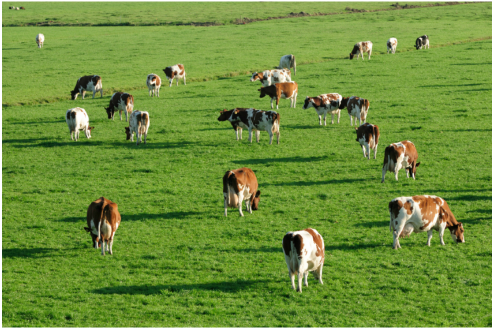


Photo credit: http://bhudeva.org/blog/2010/12/19/we-are-grazing-alot/

Concerns

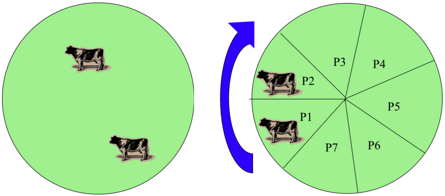


Figure: Continuous grazing (left), rotational grazing (right)

- No mathematical analysis - evidence is entirely qualitative
- Conventional advice inconsistent and approximate in terms of rotation period, cattle per acre, number of paddocks, and proper factor
- Does not consider use of multiple paddocks simultaneously

Basic Equation

$$V'(t) = G(V(t)) - H \cdot c(V(t)) \tag{1}$$

- Grazing can be modeled (Noy-Meir, 1975) (May, 1977)
  - $V(t) =$ quantity of resource
  - $G(V) =$ growth rate of resource
  - $c(V) =$ rate of consumption
  - $H =$ strength of consumption
- Previous research examines stability of multiple equilibria, not numerical solutions

Goals

- Find conditions that yield best performance for continuous grazing
- Compare productivity of rotational and continuous grazing
- Describe ideal grazing configurations that maximize, or at least obtain a balance between the number of cattle and stockpiled forage based on paddock grazing ratio and rotation period, with consideration of stability over a long period of time
- Compare this model to recommendations in application

Continuous Equation

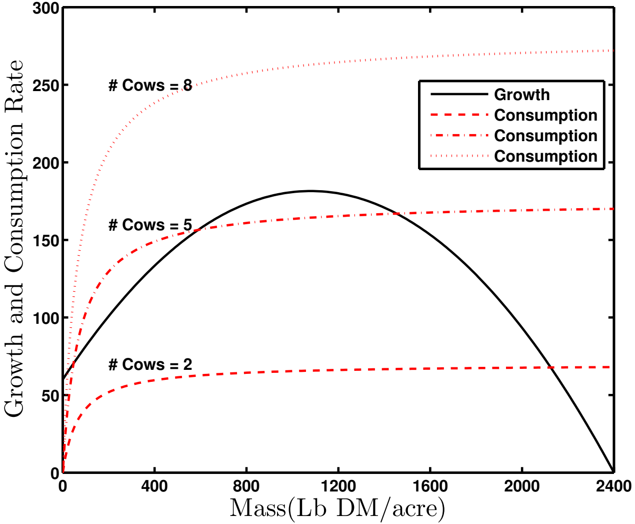
$$V'(t) = g_{max} V(t) \left(1 - \frac{V(t)}{V_{max}}\right) - H \cdot c_{max} \frac{V(t)}{V(t) + K} \qquad (2)$$

- $G(V)$ is a logistic growth function
- $c(V)$ is a Holling type II functional response (Holling, 1959)
- Solved numerically as ODE

Variable	Meaning	Units		
t	time	days		
$V(t)$	grass biomass	pounds per acre		
Constant	**Meaning**	**Units**	**Value**	**Reference**
V_{max}	grass carrying capacity	pounds per acre	2400	[5]
g_{max}	maximum growth rate per capita	day^{-1}	0.05625	[6]
c_{max}	maximum consumption rate per cattle	pounds per acre $\cdot$ day^{-1}	35	[7, 8]
K	half-saturation value	pounds per acre	120	
H	number of cattle	cattle per acre		

Table: Variables and parameters in the equations.

Example of continuous system



Equilibria

Define

$$H_{max} = \frac{g_{max}(V_{max} + K)^2}{4c_{max}V_{max}}$$

When $H < H_{max}$, there are two positive equilibria ($V'(t) = 0$). When $H > H_{max}$, there is no positive equilibrium and the grassland collapses due to overconsumption by the cattle.

- $V = 0$ is a trivial equilibrium
- V is found by plugging in the maximum H into equation (2)
- Configurations are evaluated based on maximizing H and V

Rotational Equation

$$V_j'(t) = g_{max} V_j(t) \left(1 - \frac{V_j(t)}{V_{max}}\right) - H_j(t) \cdot c_{max} \frac{V_j(t)}{V_j(t) + K}, \quad 1 \le j \le n \quad (3)$$

- V_j is the grass biomass in the jth paddock, and all constants here are same as those used for continuous grazing
- Land is divided into n equal size paddocks ($n \geq 2$)
- Forage per paddock $= \frac{2400}{n}$
- m paddocks are grazed at any single moment

Grazing Strategy

$$H_j(t) = \begin{cases} H/m, & knT + jT \le t < knT + (j+m)T, \\ 0, & knT + (j+m)T \le t < (k+1)nT + jT, \end{cases} \tag{4}$$

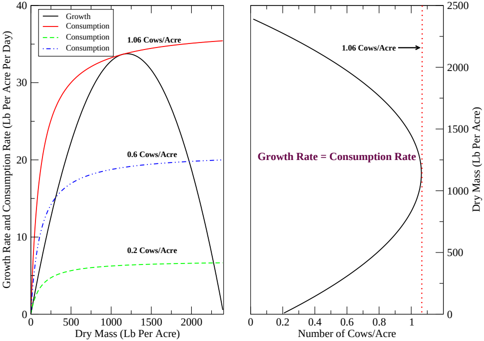
- j = label for paddock
- T = rotational period (cattle moved every T days to next paddock)
- k = any integer
- Days grazed = mT, days rested = $(n-m)T$
- Example: $n = 4$, $m = 3$, $T = 10$
  - $P_1, P_2, P_3 \longrightarrow P_2, P_3, P_4 \longrightarrow P_3, P_4, P_1 \longrightarrow P_4, P_1, P_2$
  - Each paddock grazed for $3 \times 10 = 30$ days, rested for 10 days
- Other grazing patterns are possible

Procedure for Rotational Grazing

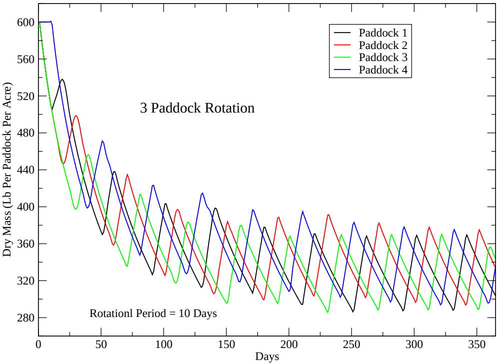
We find V under the conditions for equilibrium from continuous grazing to compute the proper factor $p = \frac{V}{V_{max}}$. This proper factor (%) is set as a minimum amount of forage that must not be grazed.

Equation (3) is solved numerically using Runge-Kutta method in MATLAB to find a maximum H per acre so that no paddock's V is below the proper factor. Excess stockpiled forage for rotational grazing is found using integration.

We first evaluate stability over 1 year and later address the situation over a longer period of time.



- The condition with 1 equilibrium state at $G(V)$'s vertex yields the largest H
- $n = m = 1$
- $H = 1.06$ cattle per acre, $V = 1140$ pounds per acre
- Proper factor $p = 1140/2400 = 47.5\%$, which is set as a baseline for rotational grazing although p may be adjusted



- $n = 4, m = 3, T = 10$
- No paddock has less than $V = \frac{2400}{n} \times p = \frac{2400}{4} \times 0.475 = 285$ pounds of forage remaining
- $H = 1.3, V = 1455.38$

Proper Factor Adjustment

When p is set to a low value such as 20%, there is only 1122.58 pounds of forage for the same conditions as before while the number of cattle increases to 1.37 heads per acre. When p is set to a higher value such as 70%, the acre can only support 0.89 cattle despite leaving 1867.21 pounds of forage available.

- Adjusting p will not increase both H and V
- If proper factor is lower for both types of grazing, rotational grazing is advantageous
- A range for the proper factor ($21\% \leq p \leq 62\%$) will increase H and V in comparison to $p = 47.5\%$ for continuous grazing
- However, $p = 47.5\%$ will be used for consistency.

Number of Cows

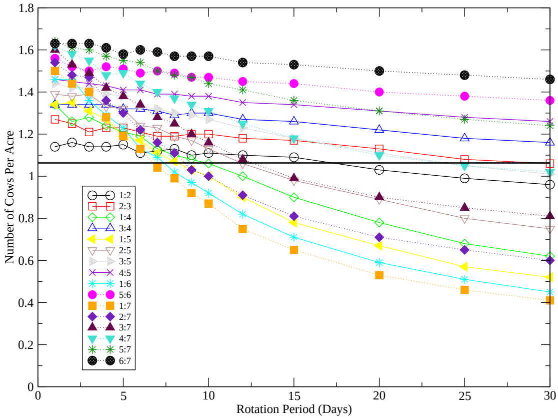


Figure: Maximum H for different paddock configurations and T, where the key is $m : n$

Amount of Forage Remaining

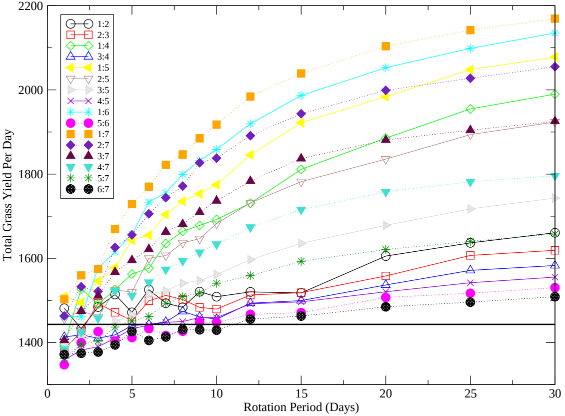


Figure: Maximum V for different paddock configurations and T

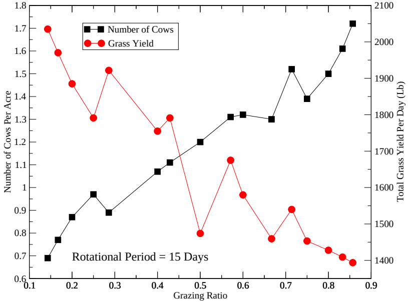
Observations

The solid line represents values obtained from continuous grazing.
As rotation period increases:

- V increases, H decreases
- More drastic changes for small $\frac{m}{n}$ ratio
- Most configurations better than continuous grazing

Finding a Balance

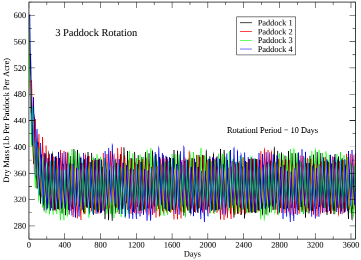
H and V appear to be inversely related. We can find a point that balances the number of cattle and the amount of forage based on the grazing ratio $\frac{m}{n}$ and the rotational period T (15 days below).



Unless one wishes to maximize H or V, a grazing ratio $\frac{m}{n} \approx 0.45$ will achieve a balance between the two variables in this situation.

Long-term Results

We have solved equation (3) only over 1 year, for which there are several schemes better than continuous grazing. What about 10 years?



- Same conditions ($n = 4, m = 3, T = 10$)
- p is still 47.5%
- $H = 1.28$ instead of 1.30, $V = 1373.78$ instead of 1455.38

Summary

- Rotational grazing is preferable in practice, but this lacks quantitative evidence
- Continuous grazing is optimized when $p = 47.5\%$, which yields $H = 1.06$, $V = 1140$
  - If $H > 1.06$, the system will collapse
- We use an ODE model and a cyclic rotational grazing strategy with no paddock containing less than 47.5% of the original amount
- With adjustment of (m, n, T), rotational configurations show an increase in H and V over continuous grazing
  - As T increases, H decreases and V increases
  - Smaller $\frac{m}{n}$ produces more drastic changes based on T
  - A grazing ratio can be found to optimize H, V, or both
  - If evaluated over a longer period of time, H and V slightly decrease

Comparisons to rotational grazing's application

What are some standards used in agriculture?

- p is recommended to be 50% in practice
- The average rotational period mT is 3 to 7 days
- The rest period $(n-m)T$ ranges from 21 to 42 days, depending on the time of year and plant type (Iowa State University Extension College of Agriculture, 2009)

We suggest the use of multiple paddocks grazed simultaneously for more productivity.

Future Considerations

- Grass type (i.e. cold and warm season grasses)
- Distribution of cattle within the individual paddocks
- Rotational grazing strategy and paddock arrangement
  - Having same $\frac{m}{n}$ grazing ratio but different n paddocks
  - Example: moving to paddock with the most forage available
  - Example: $P_1, P_2 \longrightarrow P_3, P_4$ instead of $P_1, P_2 \longrightarrow P_2, P_3$

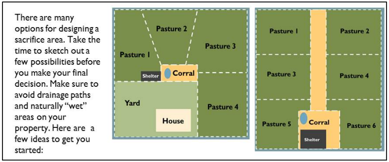
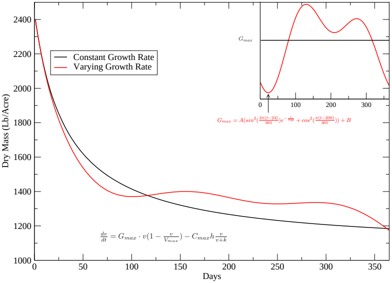


Photo Credit: http://www.ivstreamteam.org/wp-content/uploads/2012/02/sacrifice-area-layouts.jpg

Future Considerations (continued)

- We assumed that the cattle were all identical with no change in population. Instead, cattle could be modeled dynamically (i.e. different types, ages)
- A constant g_{max} is used here. However, g_{max} could vary throughout the year based on the Johnson and Thornley model of grass growth (Annals of Botany, 1983).



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